• Given two functions $f + g$, the composite function, Denoted by $f \circ g$ (read as "f composed with g"), is Defined by: $(f \circ g)(x) = f(g(x))$. The Domain of $f \circ g$ is the set of all #'s x in the Domain of g such that g(x) is in the Domain of f.

4.1 – Composite Functions

Name: ______________________________ Date:______________ Per:_____

SWBAT: 1. Form a composite function. 2. Find the domain of a composite functions

Ex.1 – Suppose that $f(x) = 2x^2 - 3$ and $g(x) = 4x$. Find the following:

a. $(f \circ g)(1)$

$f(g(1)) = f(4(1))$

$f(4) = 2(4)^2 - 3$

$= \boxed{29}$

b. $(g \circ f)(1)$

$g(f(1)) = g(2(1)^2 - 3)$

$g(-1) = 4(-1)$

$= \boxed{-4}$

c. $(f \circ f)(-1)$

$f(f(-1)) = f(2(-1)^2 - 3)$

$f(-1) = (2(-1)^2 - 3)$

$= \boxed{-1}$

Ex.2 – $f(x) = x^2 + 3x - 1$ and $g(x) = 2x + 3$. Solve and find the domain of the following:

a. $f \circ g$

$f \circ g = f(g(x)) = f(2x+3)$

$= (2x+3)^2 + 3(2x+3) - 1$

$= 4x^2 + 12x + 9 + 6x + 9 - 1$

$\boxed{f \circ g = 4x^2 + 18x + 17}$

• Since the Domains of f + g are both All real #'s, the Domain of $f \circ g$ is All real #'s.

b. $g \circ f$

$g \circ f = g(f(x)) = g(x^2 + 3x - 1)$

$= 2(x^2 + 3x - 1) + 3$

$= 2x^2 + 6x - 2 + 3$

$\boxed{g \circ f = 2x^2 + 6x - 1}$

• The Domains of f + g are both $\mathbb{R}$, so the Domain of $g \circ f$ is $\mathbb{R}$.

Ex.3 – Find the domain of $(f \circ g)(x)$ if $f(x) = 1/(x+2)$ and $g(x) = 4/(x - 1)$

$(f \circ g)(x) = f(g(x))$

$\frac{4}{x-1} = -2$

$4 = -2(x-1)$

$4 = -2x + 2$

$\frac{2}{-2} = \frac{-2x}{-2} \rightarrow \boxed{x = -1}$

• Domain of g: $\{x \mid x \neq 1\}$, so 1 must be excluded from $f \circ g$.

• Domain of f: $\{x \mid x \neq -2\}$, which means $g(x) \neq -2$

• Solve $g(x) = -2$ to determine what values of x to exclude from the Domain of $f \circ g$.

* The Domain of $f \circ g$ is $\{x \mid x \neq -1, 1\}$

Ex.4 - f(x) = 1/(x + 2) and g(x) = 4/(x – 1) Solve and find the domain of the following:

a. $g \circ f$

$$g \circ f = \frac{4}{\left(\frac{1}{x+2}\right) - 1} = \frac{4}{\frac{1 - 1(x+2)}{x+2}}$$

$$= \frac{4}{\frac{-x-1}{x+2}} = 4 \cdot \frac{x+2}{-x-1} = \frac{4(x+2)}{-(x+1)}$$

$$\boxed{g \circ f = \frac{-4(x+2)}{x+1}}$$

- Domain of f is $\{x \mid x \neq -2\}$
- Domain of g: $\{x \mid x \neq 1\}$
- Domain of $g \circ f$ is $\{x \mid x \neq -2, -1\}$

$\frac{1}{x+2} = 1 \rightarrow 1 = x + 2$

$-1 = x$

* set f(x)=1 + solve for x

b. $f \circ f$

$$f \circ f = \frac{1}{\left(\frac{1}{x+2}\right) + 2} = \frac{1}{\frac{1 + 2(x+2)}{x+2}}$$

$$= \frac{1}{\frac{2x+5}{x+2}} = \boxed{\frac{x+2}{2x+5} = f \circ f}$$

- Domain of f is $\{x \mid x \neq -2\}$
- $\boxed{\text{Domain of } f \circ f \text{ is } \{x \mid x \neq -\tfrac{5}{2}, -2\}}$
- $\frac{1}{x+2} = -2 \rightarrow 1 = -2(x+2) \rightarrow 5 = -2x$

$1 = -2x - 4$

$\frac{5}{-2} = \frac{-2x}{-2}$

$x = -\frac{5}{2}$

Ex.5 – If f(x) = 3x – 4 and g(x) = $\frac{1}{3}(x+4)$, show that $(f \circ g)(x)$ and $(g \circ f)(x) = x$, for every x in the domain of $f \circ g$ and $g \circ f$.

$$(f \circ g)(x) = f(g(x)) = f\left(\tfrac{1}{3}(x+4)\right)$$

$$= 3\left[\tfrac{1}{3}(x+4)\right] - 4$$

$$= 1(x+4) - 4$$

$$\boxed{(f \circ g)(x) = x}$$

$$(g \circ f)(x) = g(f(x)) = g(3x-4)$$

$$= \tfrac{1}{3}\left[(3x-4)+4\right]$$

$$= \tfrac{1}{3}[3x]$$

$$\boxed{(g \circ f)(x) = x}$$

* you can also verify this by putting f(x) into Y_1, g(x) into Y_2, $(f \circ g)$ into Y_3, and $(g \circ f)$ into Y_4. Use $-3 \le x \le 3$, $-2 \le y \le 2$ for viewing windows. When you press graph, you should see the line $y = x$.

Ex.6 – Finding the components of a composite function

Find the functions f and g such that $f \circ g = H$ if $H(x) = (x^2 + 1)^{50}$

- There are a few different ways to represent f(x) + g(x), so that $f \circ g = H$.
- I am going to show the most common selection for f(x) + g(x)

* Let $g(x) = x^2 + 1$ + $f(x) = x^{50}$

$$f \circ g = f(g(x)) = f(x^2+1)$$

$$= (x^2+1)^{50}$$